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Stress in piezoelectric hollow sphere with thermal gradient

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Abstract

The piezoelectric phenomenon has been exploited in science and engineering for decades. Recent advances in smart structures technology have led to a resurgence of interest in piezoelectricity, and in particular, in the solution of fundamental boundary value problems. In this paper, we develop an analytic solution to the axisymmetric problem of a radially polarized, spherically isotropic piezoelectric hollow sphere. The sphere is subjected to uniform internal pressure, or uniform external pressure, or both and thermal gradient. There is a constant thermal difference between its inner and outer surfaces. An analytic solution to the governing equilibrium equations (a coupled system of second-order ordinary differential equations) is obtained. On application of the boundary conditions, the problem is reduced to solving a system of linear algebraic equations. Finally, the stress distributions in the sphere are obtained numerically for two piezoceramics.

Keywords: Piezoelectric hollow sphere; Thermal gradient; Electric potential; Radial stress, Hoop stress

1. Introduction

In recent years, there has been an accelerated effort and notable contributions on the study of thermo-electro-elastic coupling behavior in some engineering areas, including aerospace, offshore and submarine structures, chemical vessel and civil engineering structures. These structures can be simplified to a transversely isotropic hollow sphere, and can be easily exposed to a variety of temperature fields in different environments.

The understanding of mechanical behaviors of piezoelectric structures is thus of significant importance. Because of the difficulty related to the particular coupling effect between electric field and mechanical deformation, few problems were considered before 1990. Spherical isotropy is a special kind of transverse isotropy that was introduced in 1865 by Saint Venant, who gave an exact solution of a spherically isotropic spherical shell subjected to both internal and external uniform pressures [1, 2].

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Problems of radially polarized piezoelectric bodies were considered and solved analytically [3, 4]. In the literature, the solution for isotropic medium provided static behavior such as stress concentration [3]. In previous investigations of piezoelectric structures, there are some investigations on hollow sphere. For piezoelectric materials, in [5], the static solution of radial deformation of a piezoelectric spherical shell under uniform pressures on the internal and external surfaces, and subjected to a given voltage difference between these surface, coupled with a radial distribution of temperature was successfully solved. In [6], the static solution of radial deformation of a piezoelectric cylindrical shell under uniform pressures on the internal and external surfaces, and subjected to a given voltage difference between these surface, was successfully solved.

The transient thermal stresses in a homogeneous transversely isotropic finite cylinder, due to an arbitrary internal heat generation were solved in [7]. Due to a constant temperature imposed on one surface and heat convection into the medium at the other surface, the transient thermal stresses in a homogeneous hollow cylinder were obtained in [8, 9]. Thermal shock in a hollow sphere caused by rapid uniform heating was analyzed in [10]. The dynamic thermal stresses in homogeneous isotropic solid cylinders and hollow cylinder subjected to thermal shock were studied in [11, 12].

The thermal stresses in a homogeneous, transversely isotropic, infinite cylindrical shell subjected to an instantaneous heat source were solved in [13]. The piezo-thermo-elastic behavior of a pyroelectric spherical shell was investigated [14]. An exact solution of functionally graded anisotropic cylinders subjected to thermal and mechanical loads for a steady-state problem was obtained [15]. The electro-elastic problems for a special nonhomogeneous piezoelectric hollow cylinder were studied in [16]. The nonhomogeneous material has gained much attention because of its good heat shielding character as well as other significant superiorities.

To date, investigations on the interactions of thermo electro-mechanical coupled behavior in homogeneous piezoelectric structure have mainly considered static interactions between thermal, electric and mechanical fields and transient interaction between electric field and mechanical field in a nonhomogeneous structure.

In this paper a brief summary of the thermo-electro-elastic equations for linear piezoelectric solids is given. These equations are specialized to spherical coordinates and the axisymmetric problem described above is formulated. The governing equilibrium equations in radially polarized form are shown to reduce to a coupled system of second-order ordinary differential equations for the radial displacement and electric potential field. These differential equations are solved analytically, and on applying three different sets of boundary conditions an analytic solution method for boundary value problems is developed. The stress distributions as a result of thermal difference in the hollow sphere are discussed in detail for the two piezoceramics.

2. The constitutive relation and governing equation

Consider a piezoelectric composite hollow cylinder with inner radius r_0 = a and outer radius r_2 = b. A spherical coordinate system (r,θ,φ) with the origin identical to the center of a hollow sphere is used. For the spherically symmetric problem, we have $u_\theta = u_\varphi = 0$, $u_r = u_r(r)$. For a transversely isotropic piezoelectric hollow sphere he constitutive relations of a spherically transversely isotropic pyroelectric medium are expressed as [5, 14]

$$\sigma_{rr} = c_{11}\frac{\partial u_r}{\partial r} + 2c_{12}\frac{u_r}{r} + e_{11}\frac{\partial \psi}{\partial r} - \lambda_{11}T(r) \quad (1a)$$

$$\sigma_{\theta\theta} = c_{12}\frac{\partial u_r}{\partial r} + (c_{22}+c_{23})\frac{u_r}{r} + e_{12}\frac{\partial \psi}{\partial r} - \lambda_{12}T(r) \quad (1b)$$

$$D_{rr} = e_{11}\frac{\partial u_r}{\partial r} + 2e_{12}\frac{u_r}{r} - \beta_{11}\frac{\partial \psi}{\partial r} + p_{11}T(r) \quad (1c)$$

$$\lambda_{11} = c_{11}\alpha_r + 2c_{12}\alpha_\theta,\ \ \lambda_{12} = c_{12}\alpha_r + (c_{22}+c_{23})\alpha_\theta \quad (1d)$$

where c_{ij}, e_{ij}, α_i, β_{ij}, and p_{11} are elastic constants, piezoelectric constants, thermal expansion coefficients, dielectric constants, and pyroelectric coefficients, respectively. σ_{ii} and D_{rr} are the component of stress and radial electric displacement, respectively.

The equation of equilibrium is expressed as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{2(\sigma_{rr} - \sigma_{\theta\theta})}{r} = 0 \quad (2)$$

In absence of free charge density, the charge equation of electro-statics is

$$\frac{\partial D_{rr}(r)}{\partial r} + \frac{2D_{rr}(r)}{r} = 0 \quad (3)$$

In order to simplify calculation, the following non-dimensional forms are introduced:

$$\begin{aligned}
& c_i = \frac{c_{i2}}{c_{11}}\ (i=1,2), \quad c_3 = \frac{c_{23}}{c_{11}},\ \eta = \frac{b}{a} \\
& e_i = \frac{e_{1i}}{\sqrt{c_{11}\beta_{11}}}\ (i=1,2),\ \lambda_i = \frac{\lambda_{1i}}{\alpha_r c_{11}}\ (i=1,2) \\
& p_1 = \frac{p_{11}}{\alpha_r\sqrt{c_{11}\beta_{11}}}, \quad \sigma_i = \frac{\sigma_{ii}}{\alpha_r T_0 c_{11}}\ (i=r,\theta) \\
& \phi = \sqrt{\frac{\beta_{11}}{c_{11}}}\frac{\psi}{a\,\alpha_r T_0},\ \xi = \frac{r}{a},\ u = \frac{u_r}{\alpha_r T_0 a} \\
& D_r = \frac{D_{rr}}{\alpha_r T_0\sqrt{c_{11}\beta_{11}}}, \qquad T_1(\xi) = \frac{T(r)}{T_0}
\end{aligned} \quad (4)$$

Then, Eqs. (1-3) can be rewritten as

$$\sigma_r = \frac{\partial u}{\partial \xi} + 2c_1\frac{u}{\xi} + e_1\frac{\partial \phi}{\partial \xi} - \lambda_1 T_1(\xi) \quad (5a)$$

$$\sigma_\theta = c_1\frac{\partial u}{\partial \xi} + (c_2+c_3)\frac{u}{\xi} + e_2\frac{\partial \phi}{\partial \xi} - \lambda_2 T_1(\xi) \quad (5b)$$

$$D_r = e_1 \frac{\partial u}{\partial \xi} + 2e_2 \frac{u}{\xi} - \frac{\partial \phi}{\partial \xi} + p_1 T_1(\xi) \quad (5c)$$

$$\frac{\partial \sigma_r}{\partial \xi} + \frac{2(\sigma_r - \sigma_\theta)}{\xi} = 0 \quad (6a)$$

$$\frac{\partial D_r(\xi)}{\partial \xi} + \frac{2D_r}{\xi} = 0 \quad (6b)$$

where, a and b are the inner and outer radii of the hollow sphere, respectively, and T_0 is the reference temperature. From Eq. (6b), we have

$$D_r(\xi) = \frac{A_1}{\xi^2} \quad (7)$$

where A_1 is a constant. Substituting Eq. (7) into Eq. (5c), gives

$$\frac{\partial \phi}{\partial \xi} = e_1 \frac{\partial u}{\partial \xi} + 2e_2 \frac{u}{r} - \frac{A_1}{\xi^2} + p_1 T_1(\xi) \quad (8)$$

Utilizing Eq. (8), Eqs. (5a) and (5b) may rewritten as

$$\sigma_r = (1+e_1^2)\frac{\partial u}{\partial \xi} + 2(c_1 + e_1 e_2)\frac{u}{\xi} - \frac{e_1}{\xi^2} A_1 - (\lambda_1 - e_1 p_1) T_1(\xi) \quad (9a)$$

$$\sigma_\theta = (c_1 + e_1 e_2)\frac{\partial u}{\partial \xi} + (c_2 + c_3 + 2e_2^2)\frac{u}{\xi} - \frac{e_2}{\xi^2} A_1 - (\lambda_2 - e_2 p_1) T_1(\xi) \quad (9b)$$

Substituting Eqs. (9) into Eq. (6a), the basic displacement equation of a transversely isotropic piezoelectric hollow sphere is expressed as

$$\frac{\partial^2 u(\xi)}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial u(\xi)}{\partial \xi} - \frac{H^2 u(\xi)}{\xi^2} = I\frac{A_1}{\xi^3} + g(\xi) \quad (10a)$$

Where

$$H^2 = \frac{2(c_2 + c_3 + 2e_2^2 - c_1 - e_1 e_2)}{m}, \quad I = -\frac{2e_2}{m}, \quad m = 1 + e_1^2, \quad g(\xi) = \left[A\frac{\partial T_1}{\partial \xi} + B\frac{T_1}{\xi}\right] \quad (10b)$$

$$A = \frac{1}{m}(\lambda_1 - e_1 p_1)$$

$$B = \frac{2}{m}(\lambda_1 - \lambda_2 + (e_2 - e_1) p_1)$$

The heat conduction equation for hollow sphere can be written as

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{dT}{d\xi}\right) = 0 \quad (11)$$

The solution to this equation can be written as

$$T = k_2' - \frac{k_1'}{\xi}$$

$$\left.\begin{aligned} \xi = 1 \Rightarrow T = T_a \\ \xi = \eta \Rightarrow T = T_b \end{aligned}\right\} \Rightarrow \begin{cases} k_1' = \dfrac{\eta}{\eta - 1}(T_b - T_a) \\ k_2' = k_1' + T_a \end{cases} \quad (12a)$$

where T_a and T_b are temperature in the inner and outer surface of hollow sphere, respectively. It is assumed that $T_a = T_0$ so T_1 can be derived as

$$T_1(\xi) = k_2 - \frac{k_1}{\xi}, \quad k_1 = \frac{\eta}{\eta - 1}\left(\frac{T_b}{T_0} - 1\right), \quad k_2 = k_1 + 1 \quad (12b)$$

Since T_1 is known the solution to Eq. (10a) can be written as

$$u = F_1 \xi^{\alpha_1} + F_2 \xi^{\alpha_2} + \frac{Bk_2}{2 - H^2}\xi - \frac{IA_1}{H^2 \xi} + \frac{k_1(B - A)}{H^2} \quad (13a)$$

Where F_1, F_2 and A_1 are constant, and

$$\alpha_{1,2} = \frac{-1 \pm \sqrt{1 + 4H^2}}{2} \quad (13b)$$

Since u(r) is known, the electrostatic potential is obtained from Eq. (8)

$$\phi = F_1\left(e_1 + \frac{2e_2}{\alpha_1}\right)\xi^{\alpha_1} + F_2\left(e_1 + \frac{2e_2}{\alpha_2}\right)\xi^{\alpha_2} + \frac{A_1}{\xi}\left(\frac{2e_2 I}{H^2} - \frac{e_1 I}{H^2} + 1\right) + A_2 + L_1 \quad (14a)$$

Where

$$L_1 = \frac{Bk_2}{2-H^2}\left(e_1 + 2e_2\right)\xi + \frac{k_1\left(B-A\right)}{H^2}\left(e_1 + 2e_2 \ln \xi\right) + p_1\left(k_2\xi - k_1 \ln \xi\right) \quad (14b)$$

Then, Eq. (9a) can be rewritten as

$$\sigma_r = F_1\left(\left(1+e_1^2\right)\alpha_1 + 2\left(c_1 + e_1e_2\right)\right)\xi^{(\alpha_1-1)} + F_2\left(\left(1+e_1^2\right)\alpha_2 + 2\left(c_1 + e_1e_2\right)\right)\xi^{(\alpha_2-1)} + \frac{A_1}{\xi^2}\left(\frac{I}{H^2}\left(1+e_1^2 - 2\left(c_1+e_1e_2\right)\right) - e_1\right) + L_2 \quad (15a)$$

Where

$$L_2 = \left(1+e_1^2\right)\left(\frac{Bk_2}{2-H^2}\right) + 2\left(c_1+e_1e_2\right)\left(\frac{Bk_2}{2-H^2} + \frac{k_1\left(B-A\right)}{H^2\xi}\right) - \left(\lambda_1 - e_1p_1\right)T_1\left(\xi\right) \quad (15b)$$

Then, Eq. (9b) can be rewritten as

$$\sigma_\theta = F_1\left(\left(c_1+e_1e_2\right)\alpha_1 + \left(c_2+c_3+2e_2^2\right)\right)\xi^{(\alpha_1-1)} + F_2\left(\left(c_1+e_1e_2\right)\alpha_2 + \left(c_2+c_3+2e_2^2\right)\right)\xi^{(\alpha_2-1)} + \frac{A_1}{\xi^2}\left(\frac{I}{H^2}\left(c_1+e_1e_2-\left(c_2+c_3+2e_2^2\right)\right) - e_2\right) + L_3 \quad (16a)$$

Where

$$L_3 = \left(c_1+e_1e_2\right)\left(\frac{Bk_2}{2-H^2}\right) + \left(c_2+c_3+2e_2^2\right)\left(\frac{Bk_2}{2-H^2} + \frac{k_1\left(B-A\right)}{H^2\xi}\right) - \left(\lambda_2 - e_2p_1\right)T_1\left(\xi\right) \quad (16b)$$

Three sets of boundary conditions, henceforth referred to as cases 1, 2, 3, are examined. In case 1, the sphere is subjected to an internal uniform pressure, zero electric potential difference across the spherical annulus, and free mechanical boundary conditions on the outer surface. In this case, the sphere acts as a sensor. In the second case, free mechanical boundary conditions on both internal and external surfaces were imposed. However, there is a uniform potential difference prescribed across the annulus. In this case, the sphere acts as an actuator. For convenience, it is assumed that the potential on the outer surface is zero, and the potential on the inner surface is a nonzero constant. In the third case, free mechanical boundary conditions on both internal and external surfaces were imposed. However, there is a uniform potential difference prescribed across the annulus. For convenience, it is assumed that the potential on the inner surface is zero, and the potential on the outer surface is a nonzero constant.

The boundary conditions for each case can be written as follows:

$$\begin{aligned} &case1: \quad \sigma_r(1) = -P_i, \quad \sigma_r(\eta) = 0, \quad \phi(1) = 0, \quad \phi(\eta) = 0 \\ &case2: \quad \sigma_r(1) = 0, \quad \sigma_r(\eta) = 0, \quad \phi(1) = \phi, \quad \phi(\eta) = 0 \\ &case3: \quad \sigma_r(1) = 0, \quad \sigma_r(\eta) = 0, \quad \phi(1) = 0, \quad \phi(\eta) = \phi \end{aligned} \quad (17a)$$

For simplicity the boundary conditions are normalized as: P_i=1 and φ = 1; therefore, the boundary conditions can be written as

$$\begin{aligned} &case1: \quad \sigma_r(1) = -1, \quad \sigma_r(\eta) = 0, \quad \phi(1) = 0, \quad \phi(\eta) = 0 \\ &case2: \quad \sigma_r(1) = 0, \quad \sigma_r(\eta) = 0, \quad \phi(1) = 1, \quad \phi(\eta) = 0 \\ &case3: \quad \sigma_r(1) = 0, \quad \sigma_r(\eta) = 0, \quad \phi(1) = 0, \quad \phi(\eta) = 1 \end{aligned} \quad (17b)$$

For each of these cases, the system of linear algebraic equations for the constants F_1, F_2, A_1 and A_2 can be written in the form

$$Ma_n = b_n - b_0 \qquad (n = 1,2,3) \quad (18)$$

Where

$$b_0 = \begin{bmatrix} L_2\big|_{\xi=1} \\ L_2\big|_{\xi=\eta} \\ L_1\big|_{\xi=1} \\ L_1\big|_{\xi=\eta} \end{bmatrix} \quad (19)$$

Where the coefficient matrix M is defined in terms of column vectors:

$$M = \left[m_1\, m_2\, m_3\, m_4\right] \quad (20)$$

Where

$$m_1 = \begin{bmatrix} (1+e_1^2)\alpha_1 + 2(c_1 + e_1e_2) \\ ((1+e_1^2)\alpha_1 + 2(c_1 + e_1e_2))\eta^{(\alpha_1-1)} \\ e_1 + \dfrac{2e_2}{\alpha_1} \\ \left(e_1 + \dfrac{2e_2}{\alpha_1}\right)\eta^{\alpha_1} \end{bmatrix} \quad (21a)$$

$$m_2 = \begin{bmatrix} (1+e_1^2)\alpha_2 + 2(c_1 + e_1e_2) \\ ((1+e_1^2)\alpha_2 + 2(c_1 + e_1e_2))\eta^{(\alpha_2-1)} \\ e_1 + \dfrac{2e_2}{\alpha_2} \\ \left(e_1 + \dfrac{2e_2}{\alpha_2}\right)\eta^{\alpha_2} \end{bmatrix} \quad (21b)$$

$$m_3 = \begin{bmatrix} \left(\dfrac{I}{H^2}(1+e_1^2 - 2(c_1 + e_1e_2)) - e_1\right) \\ \dfrac{1}{\eta^2}\left(\dfrac{I}{H^2}(1+e_1^2 - 2(c_1 + e_1e_2)) - e_1\right) \\ \left(\dfrac{2e_2I}{H^2} - \dfrac{e_1I}{H^2} + 1\right) \\ \dfrac{1}{\eta}\left(\dfrac{2e_2I}{H^2} - \dfrac{e_1I}{H^2} + 1\right) \end{bmatrix} \quad (21c)$$

$$m_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad (21d)$$

Each set of boundary conditions determines the form of the column vector b_n on the right hand of equation since $Ma_n = b_n - b_0 \ (n = 1,2,3)$,
Where

$$b_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (22)$$

and

$$a_n = \begin{bmatrix} F_{1n} \\ F_{2n} \\ A_{1n} \\ A_{2n} \end{bmatrix} \quad (23)$$

The unknown constants F_1, F_2, A_1 and A_2 have been obtained by Cramer's rule, and accordingly:

$$F_{1n} = \frac{|M_{1n}|}{|M|} \quad F_{2n} = \frac{|M_{2n}|}{|M|}$$
$$A_{1n} = \frac{|M_{3n}|}{|M|} \quad A_{2n} = \frac{|M_{4n}|}{|M|} \quad (24)$$

Where

$$\begin{aligned} M_{1n} &= [b_n - b_0 \quad m_2 \quad m_3 \quad m_4] \\ M_{2n} &= [m_1 \quad b_n - b_0 \quad m_3 \quad m_4] \\ M_{3n} &= [m_1 \quad m_2 \quad b_n - b_0 \quad m_4] \\ M_{4n} &= [m_1 \quad m_2 \quad m_3 \quad b_n - b_0] \end{aligned} \quad (25)$$

3. Numerical results and discussions

The numerical results are drawn in diagrams showing the variation of stress and potential across the thickness of the hollow sphere. Mechanical and electrical and thermal properties of piezoelectric materials are listed in Table 1 [17].

The plots in the figures depict results for each of the boundary conditions, with different aspect ratio, η=1.6, 1.8, 2 .The plots in the figures depict results for $T_a=T_0$ and $T_b=3T_0$. All quantities are plotted versus dimensionless radius $\xi = \frac{r}{a}$. Since $1 \le \xi \le \eta$, the plot for a given aspect ratio will terminate at a respective value of η . In Figs. 1, 2, 3 and 4, the piezoceramic number (1), is shown by solid line and the piezoceramic number (2) by dashed line.

Table 1. Piezoelectric property.

Property	Number(2)	Number(1)
C_{11} (all GPa)	111	139
C_{12}	77.8	77.8
C_{13}	74.3	74.3
C_{22}	125.6	139
C_{33}	111	115
C_{23}	74.3	74.3
e_{11} (all c/m^2)	15.1	-5.2
e_{12} e_{13}	-5.2 -5.2	15.1 -5.2
α_r(1/k)	2×10^{-5}	2×10^{-5}
α_θ(1/k)	2×10^{-6}	2×10^{-6}
P_{11}(C/m^{2}k)	-2.5×10^{-5}	-2.5×10^{-5}
β_{11}(C^2/Nm^2)	5.62×10^{-9}	3.87×10^{-9}

In Fig. 1, results are shown for case 1, where internal pressure is applied. The compressive radial stresses plotted in Fig. 1(a) are interestingly maximum in the interior surface. As the aspect ratio η increases to 2, the maximum compressive radial stress shifts to the inner radius. The hoop stress shown in Fig. 1(b) decreases from the inner to the outer radius. The magnitude of hoop stress increases with increasing aspect ratio for piezoceramic number (2). The compressive hoop stresses are interestingly minimum in the interior surface for piezoceramic number (1). Fig. 1(c) shows the resulting induced electrical effect, and an electric potential has developed through the thickness of the sphere.

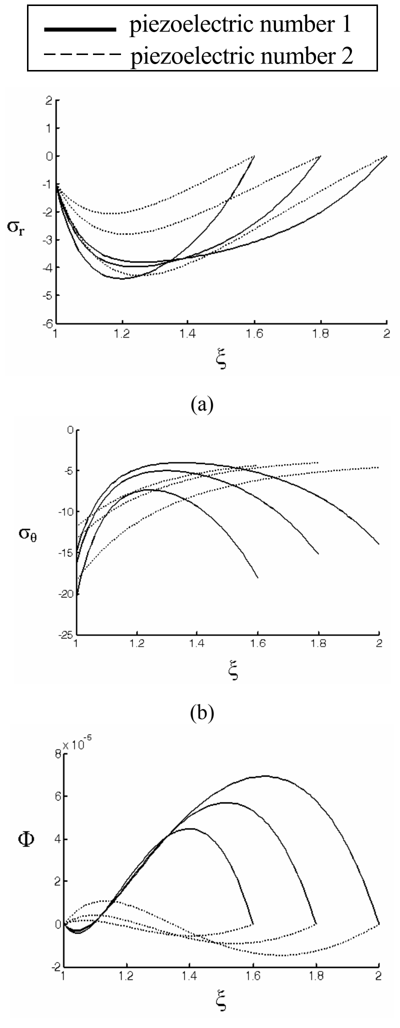

Fig. 1. Case 1: Plots for stresses and potential for η = 1.6, 1.8, 2 and T_b/T_a=3.

Fig. 2 shows the results of case 2 (purely electrical) boundary conditions. The tensile radial stresses plotted in Fig. 2(a) are interestingly maximum in the interior surface. As the aspect ratio η , increases to 2, the maximum radial stress shifts to the inner radius. The piezoceramic number (2) has greater stresses than piezoceramic number (1). The hoop stress shown in Fig. 2(b) decreases from the inner to the outer radius. The magnitude of hoop stress increases with increasing aspect ratio for piezoceramic number (2), and the piezoceramic number (2) has greater stresses than piezoceramic number (1). Its magnitude decreases with increasing aspect ratio, and its value at the middle radius approaches zero for large aspect ratios for piezoceramic number (1).

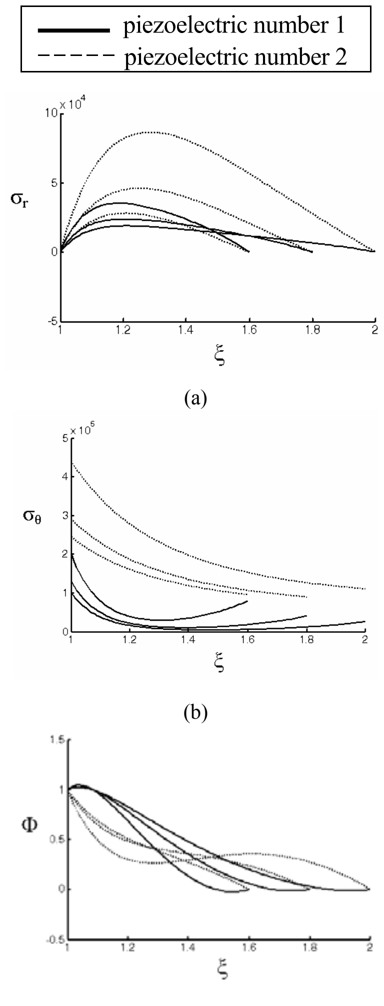

Fig. 2. Case 2: Plots for stresses and potential for η = 1.6, 1.8, 2 and T_b/T_a=3.

Fig. 3 shows the results of case 3. Stress in this case change with changing electric potential field direction in comparison with the previous case. The compressive radial stresses plotted in Fig. 3(a) are interestingly maximum in the interior surface. The piezoceramic number (2) has greater stresses than piezoceramic number (1). The hoop stress shown in Fig. 3(b) decreases from the inner to the outer radius. The magnitude of hoop stress increases with increasing aspect ratio for piezoceramic number (2). Its magnitude decreases with increasing aspect ratio, for piezoceramic number (1).

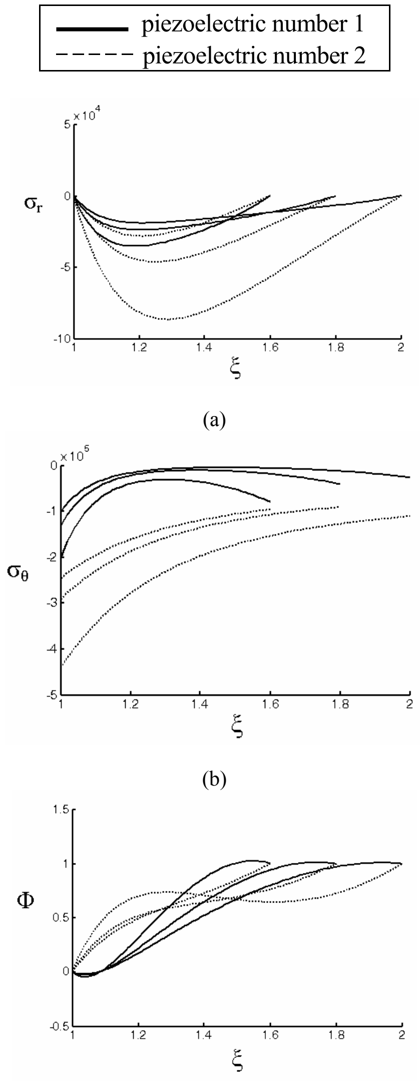

Fig. 3. Case 3: Plots for stresses and potential for η = 1.6, 1.8, 2 and T_b/T_a=3.

4. Conclusions

In this research the static behavior of radially polarized piezoelectric hollow spheres was studied and the following results were concluded:

Thermal gradient can vary radial stress and hoop stress in a piezoelectric hollow sphere.

In geometrically symmetric shapes, e.g., a piezoelectric sphere that can be polarized in radial direction, the mechanical and electrical effects can be investigated separately. The analysis approach presented in this research can be applied on all radially polarized piezoelectric spheres.

A solution to the problem of static radial displacement and potential field of a piezoelectric spherically isotropic hollow sphere with thermal gradient, polarized in the radial direction for three different sets of boundary conditions was obtained.

Dimensionless stress distributions and electric potential curves were drawn and discussed in detail for two piezoceramics, and the effects of different boundary conditions as well as piezoelectric materials on the stress state in piezoelectric hollow sphere were studied.

The hoop stress compared to the radial stress causes failure of the elastic hollow spheres. For two piezoceramics, at the second loading case (Fig. 2), hoop stress distribution on internal surface of the sphere is tensile for each aspect ratio which provides an appropriate location for fatigue crack growth.

The technological implications of this study are significant, e.g., the amount of hoop stress resulting from mechanical loads in a hollow piezoelectric cylinder can be reduced or neutralized by a suitably applied electrical field.

Nomenclature

ε_{ij} : Component of strains
u_r : Radial displacement [m]
c_{ij}, e_{ij}, α_i, p_{ij} : Elastic constants [N/m^2], piezoelectric constants [C/m^2], thermal expansion coefficients [1/k], and dielectric constants [C^2/Nm^2]
λ_i, β_{ii} : Thermal modulus [N/m^2 K], and pyroelectric coefficient [C/m^{2}K]
σ_{ij}, D_{rr} : The component of stresses [N/m^2] and radial electric displacement [C/m^2]
$\psi(r)$: Electric potential [W/A]
$T(r)$: Temperature change function [k]
r : Radial variable [m]

a, b : Inner and outer radii of piezoelectric hollow sphere [m]

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